Low Energy Interplanetary Transfers Using the Invariant Manifolds of L1, L2, and Halo Orbits

Martin W. Lo
Shane D. Ross
Jet Propulsion Laboratory
California Institute of Technology

Abstract

The invariant manifolds associated with the outer planets are extremely large objects in phase space. They are trajectories in the ecliptic which intersect one another. The enables a low energy single impulse transfer between the planets which requires several orbital periods. However, if we consider the Jovian satellites where the same dynamics occur but with much shorter orbital periods, this approach may be used for new tour designs requiring minimal ΔV. The existence of this transfer is an indication of the instability of the region of space between the satellites. It may explain some of the difficulties encountered in traditional satellite tour designs using conic approximations.

Introduction

This is one of a planned series of papers describing work which began in 1994 to explore the dynamics of the three body problem to find new trajectories for future missions. The discoveries made in this work will open up new families of low-energy missions. Some of the ideas already being used are:

1. The design of the Genesis Discovery Mission orbit (Ref. Howell, Barden, Lo).
2. The ARIANE V Piggy Back Option to Mars and Venus.

It contributes to long-standing phenomena in the dynamics of the Solar System and celestial mechanics including:

1. The Temporary Capture of Jupiter Comets.
2. The Stability of the Hilda Asteroids.
3. The Kirkwood Gaps.
4. The Structure of the Zodical Dust Tori
5. Interplanetary Transport Between The Kuiper Belt and The Inner Solar System

The major theme in this work is the interplay between Solar System dynamics and astrodynamics. Nature's resourcefulness and efficiency are hard to beat. By understanding the motion of natural bodies, that same dynamics can be used for space missions. For example the Temporary Capture of comets by Jupiter suggests a low energy capture of a spacecraft by a planet or moon. Similarly, the same mean motion resonances which govern the asteroid distribution also provide a map for low energy interplanetary transfers.

The second theme in this work is the use of dynamical systems theory to achieve low energy transfers. This rich and complex dynamics is provided gratis by the collinear Lagrange points L1, L2, and L3.

In order to understand the applications to space missions, we must first understand the natural dynamics of the Solar System which we want to use. This paper focuses on the interplanetary transport and the temporary capture mechanisms controlled by L1 and L2. The reader is referred to the rich but difficult paper of Llbre, Martinez, and Simo for an in depth description of the mathematical foundations.

The Dynamic Currents In Space

We typically think of space as empty. But dynamically, it is actually filled with layers and layers of high dimensional energy surfaces. A particle on an energy surface is constrained to move on that surface until its energy changes such as through a collision or propulsion. The energy is itself foliated (filled) with layers and layers of lower dimensional surfaces which in one way or another constrain the motion of particles on them. These surfaces are called invariant surfaces and space is foliated with them much like an onion with layers and layers of skins. Ultimately, they are made up of trajectories in the space. The mathematical name for the invariant surfaces is *invariant manifolds*.

Among the invariant manifolds are two important classes called the *stable manifold* and the *unstable manifold* which are associated to unstable periodic orbits. The stable manifold consists of trajectories which exponentially approach the periodic orbit. The unstable manifold consists of trajectories which exponentially leaves the periodic orbit. These manifolds are like the great currents of the ocean or the jet stream in the atmosphere which are able to quickly convey objects from one region of space to a distant region. In the case of space trajectories, these manifolds can provide an essentially zero-ΔV transport over vast regions!

Today's trajectory design methodology is akin to ancient mariners crossing the Atlantic with little or no knowledge of the major currents driving the ocean. Sometimes they are trapped in the still waters of the Zaragosa sea; sometimes they fortuitously tap into a channel which sends them speedily to their target ;sometimes they are thrown way off course not knowing that the currents have changed. We, like them, are searching for a course to navigate in space without knowledge of the great dynamical currents and channels in space. Frequently we run into energy barriers without a good understanding of how they arise or how to get around them. Sometimes we get lucky and find an incredibly low energy transfer which can be difficult to reproduce if initial conditions are changed even slightly. Much of trajectory design today depends on the collected wisdom and knowledge of old space mariners which are handed down in an oral tradition not unlike that of sea faring days of yore.

The stable and unstable manifolds are some of the most powerful "gravitational jet streams" in the Solar System which govern the motions of the celestial bodies and shaped the structure of the Solar System. They provide low energy transfers and planetary captures which are of great interests to future missions. Knowledge of this system of currents and channels which connect the entire Solar System are essential for mission design in the new millenium.

The Source of the Dynamical Currents and Channels: The Lagrange Points

Figure 1a shows the five Lagrange points in the Sun-Jupiter system in rotating coordinates. The origin is at the Sun-Jupiter barycenter; the Sun and Jupiter are fixed on the x-axis in this coordinate system with Jupiter's orbit in the xy-plane. The most fundamental invariant manifolds of this system are associated with the five Lagrange points. They are the seeds of the dynamics of this system. We will concentrate on the role of L1 and L2.

L1 and L2 each have a one-dimensional stable and unstable manifold. In Figure 1a, the manifolds of L1 are plotted in green; the manifolds of L2 are plotted in black. The stable manifold is indicated by the dashed curve, the unstable manifold by the solid curve. The strange shapes of these curves are the result of the rotating coordinate system which is evoke these patterns that are otherwise ordinary-looking elliptical orbits in inertial space. This, in part, is the reason rotating coordinates are so powerful for this problem. We divide space into three regions: the region inside of Jupiter's orbit (dashed circle about the Sun) is the Interior Region; the region outside of Jupiter's orbit is the Exterior Region; the region between L1 and L2 which includes Jupiter is the Capture Region for reasons which will be clear shortly. Figure 1b shows the manifolds in the Capture Region where they are "captured" by Jupiter. Note that the stable manifold of L1 and the unstable manifold of L2 are very near each other and conspire to move from L2 to L1 showing that there is a dynamical connection between L2 and L1. A similar connection exists between L1 and L2 but the additional manifolds would confuse the already complex picture and are therefore not shown here.

If L1 and L2 are the seeds of this dynamical system, the invariant manifolds of L1 and L2 are like the DNA of the system in the following sense. Their behavior characterizes that of the system; by understanding how these four trajectories, the stable and unstable manifolds of L1 and L2, behave, we can get an incredibly detailed picture of many features of the Solar System. Knowledge of this dynamics in turn allows us to design new trajectories to enable the increasingly more demanding new mission concepts.

Applications to Solar System Dynamics

1. The Temporary Capture of Comets by Jupiter

In Figure 2a we have superimposed the orbit of the comet Oterma on that of Figure 1. Figure 2b is a blow up of Jupiter's Capture Region. Recall everything is plotted in rotating coordinates. Oterma's orbit is obtained from JPL Section 312's Horizon 2000 small body trajectory integrator. Note how closely the comet orbit follows the manifolds going into the Capture Region and exiting it in Fig. 2b. Note how the comet orbit in the Interior Region first follows the unstable manifold leaving L1, but then gets caught on the stable manifold returning it to L1. In both the Interior and Exterior Regions, the comet orbit has the characteristic shapes of the manifolds in the corresponding region.

Figure 3a,b is the same portrait for the comet Gehrels 3. In this case, the comet is captured by Jupiter temporarily for several orbits. This is the so-called Temporary Capture Phenomenon for Jupiter family comets. Note there is a kidney-shaped orbit around L2 which is typical of halo orbits. Halo orbits are well known periodic orbits around the Lagrange points which have invariant manifold structures of their own. Evidently, Gehrels 3 was attracted by the stable manifold of this halo orbit and then repelled by its unstable manifold resulting in this picture.

Figures 4a,b provide the protrait of comet Helin-Crockett-Roman. Like Gehrels 3, it too was temporarily captured by Jupiter for several periods. A careful estimate of the periods of all three comet orbits as well as that of the manifolds of L1 and L2 show that in the Interior Region, the orbits are all near the 3:2 resonance of Sun-Jupiter system; in the Exterior Region, the orbits are all near the 2:3 resonance. (By 3:2 resonance we mean the comet travels 3 times around the Sun for every two orbits of Jupiter around the Sun.) Thus, by studying the stable and unstable manifolds of L1 and L2, we can get a sense of how the temporary capture phenomenon occurs. This includes the fact that the comet must enter and exit by way of L1 and L2 into and out of the Capture Region; the comet orbit changes resonance period going between the Interior and Exterior Regions and always in the characterisitic 3:2 and 2:3 resonances. These facts are well documented by observations and dynamical simulations (Ref. Belbruno & Marsden). They are explained by the invariant manifolds of L1 and L2.

It is remarkable that two simple orbits, the stable and unstable manifold of Jupiter's L1, have provided such a wealth of information on the structure of the Solar System just from cursory examination of their geometry. They are, in this sense, the DNA of the dynamics of the Solar System.

2. *SURFing*: Interplanetary Transport

The manifolds of the planets intersect! Figure 5 shows the intersection of Jupiter's L2 manifolds and Saturn's L1 manifolds. This case is special because it occurs so quickly and because the two manifolds are nearly in phase. This may have something to say about the so-called Great Inequality, the near commensurability of their periods. Also, the fact that the intersection occurs so quickly is an indication that transport between Jupiter and Saturn can occur easily so that we would not expect to find any permament belt structure in this region. However, the more important thing to notice is that their manifolds intersect. In fact from Jupiter to Neptune, the manifolds of all of the outter planets' L1 and L2 manifolds intersect one another forming a great gravitational network of dynamic currents and channels between the Asteroid Belt and the Kuiper Belt. For the Inner Solar System, the connection between Earth and Venus has been verified. Verification for the other cases is underway.

In other words, great portions of the entire Solar System is dynamically linked by these manifolds. This offers a transport mechanism whereby objects can move in and out of the Solar System along this pathway of L1 and L2 manifolds. In the current configuration, a change in velocity is required to transition from one manifold to the next. This can occur from collisions, for example. However, nearby paths exists which do not require a ΔV much as the orbit of Oterma transitioned from the unstable manifold of L1 onto the stable manifold of L1 in the Interior Region of Jupiter without a ΔV. In fact, as mentioned before, the manifolds of L1 and L2 are the "seed" orbits. There are layers and layers of manifolds of the halo and lissajous orbits which foliate the space centered on the manifolds of L1 and L2. These complex manifolds are really the ones governing the dynamics known as "lobe dynamics arising from homoclinic orbits" which was just discovered in the last 10 years. An excellent reference is Wiggins.

~~Figure 6 shows~~ The intersection of the L2 unstable manifold of Jupiter with the L1 stable manifold of Saturn in Fig 5. ~~This~~ provides a transfer from Jupiter's L2 to Saturn's L1 which required 13 years time of flight and a ΔV of 900 m/s at the intersection of the manifolds. Compare this with the standard Hohmann transfer which requires 2716 m/s and 9.9 years. While this orbit may only be of academic interest due to the long time of flight, the same technique applies to planet-moon systems where the periods are much smaller so that such transfers become useful for mission design purposes.

In addition to the transport, the Lagrange points can also be used to provide low energy captures such as was demonstrated by the temporary capture of Jupiter comets. Indeed this technique was used to rescue the Japanese Hiten Mission with great success [Ref. Belbruno, Miller]

We think of the gravitational network of dynamical currents as a series of waves provided by the planetary tidal forces on which a comet or spacecraft can surf from planet to planet, stopping occasionally in a temporary capture, making the Grand Tour on an economy ΔV-budget.

Summary

The network of dynamic currents and channels generated by the stable and unstable manifolds of the planetary L1 and L2 Lagrange points play an important role in the distribution and transport of material in the Solar System. We have shown that they guide the paths of Jupiter family comets in their temporary capture by Jupiter. Like the terzarima rhyme scheme which unifies Dante's "Divine Comedy", the gravitational network links planet to planet providing a transport mechanism through the entire Solar System from the Sun to the Kuiper Belt.

In a subsequent paper we will show how these simple 1-dimensional manifolds enable us to visualize the complex resonance structure of the Solar System. Using more refined techniques we will be able to compute transport rates and density distributions which are currently under way. They provide a remarkably simple, unified explanation for many disparate dynamical phenomena in the Solar System on every scale from comet motion to belt structures and dust tori.

Remarks

This work began with a SURF project in 1994 which is why we chose "*SURFing*" to name this approach.

Acknowledgments

The authors wish to thank Peter Goldreich, Gerard Gomez, Angel Jorba, Josep Masdemont, Scott Tremaine, and Stephen Wiggins for the conversations and exchanges regarding this problem. The wonderful tools provided by Gomez and Masdemont greatly speeded the computation of the manifolds. Special thanks are due to Jerrold Marsden and Ralph Roncoli for their support and careful review of this paper, and to Andrew Lange who first got us together. Lastly, a special acknowledgment to Kathleen Howell long-term collaboration with the first author has added immeasureably to our work. This work was carried out at the Jet Propulsion Laboratory, California Institute of Technology for the NASA Advanced Concepts Research Program, "Applications of Dynamical Systems Theory to the Design and Development of Spacecraft Trajectories", under a contract with the National Aeronautics and Space Administration, the Caltech Summer Undergraduate Research Fellowship Program, and the Caltech Work Study Program.

References

Belbruno, E., Marsden, B., "Resonance Hopping in Comets", The Atronomical Journal 113(4), April 1997, 1433-1444

Belbruno, E. and Miller, M., "Sun-Perturbed Earth-to-Moon Transfers with Ballistic Capture", J. of Guidance, Control, and Dynamics, v. 16 (1993), 770-775

J. D. Giorgini, D. K. Yeomans, A.B. Chamberlin, P.W. Chodas, R.A. Jacobson, M.S. Keesey, J.H. Lieske, S.J. Ostro, E.M. Standish, R.N. Wimberly, "Horizons JPL's On-Line Solar System Data and Ephemeris Computation Service - Any Body - Anytime", JPL document, October 1996

C. Howell, B. Barden, M. Lo, "Application of Dynamical Systems Theory to Trajectory Design for a Libration Point Mission", American Astrodynamics Journal, submitted

Llbre, J., Martinez, R., and Simo, C., Transversality of the Invariant Manifolds Associated to the Lyapunov Family of Periodic Orbits Near L2 in the Restricted Three-Body Problem, Journal of Differential Equations 58 (1985), 104-156

Wiggins, S., Global Dynamics, Phase Space Transport, Orbits Homoclinic to Resonances, and Applications, Fields Institute Monographs, American Mathematical Society, 1993

Figure 1a. Jupiter L1 (green) and L2 (black) manifolds in Sun–Jupiter barycentered rotating frame

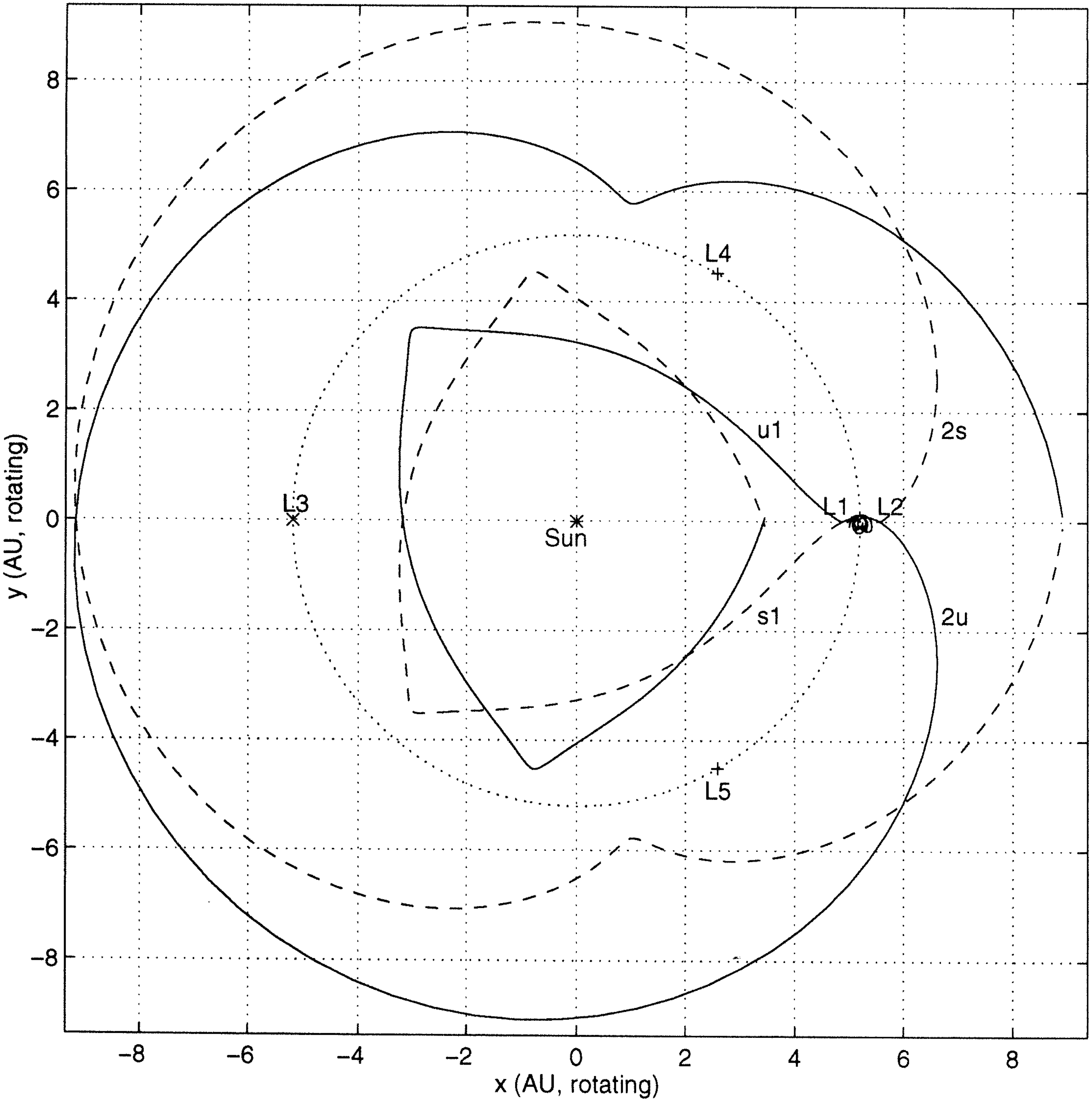


Figure 1b. Jupiter L1 and L2 manifolds near Jupiter

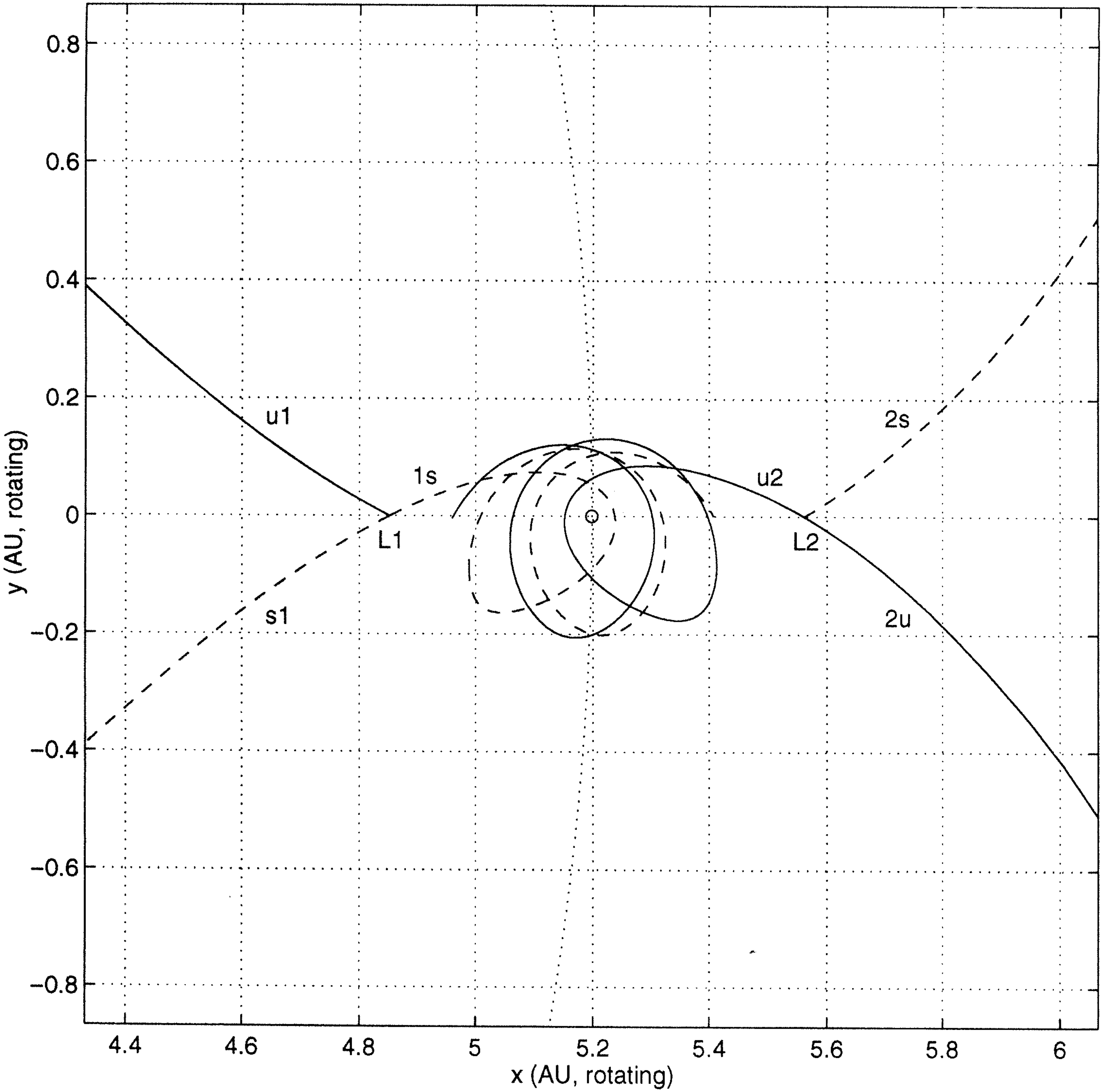
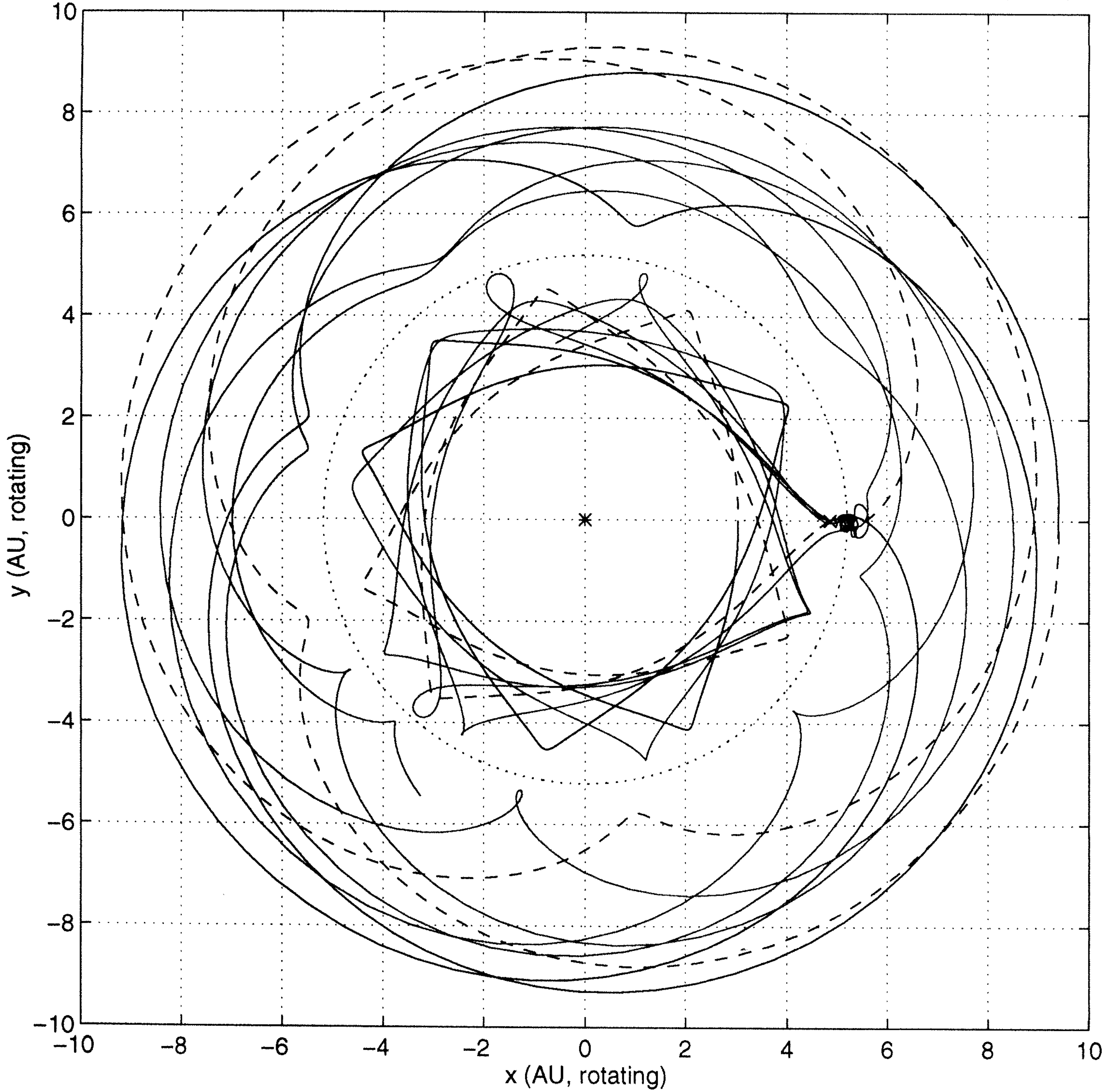


Figure 2a. comet Gehrels 3 (red) from 1800–2100 AD with Jup us1 (green) and 2su (black)



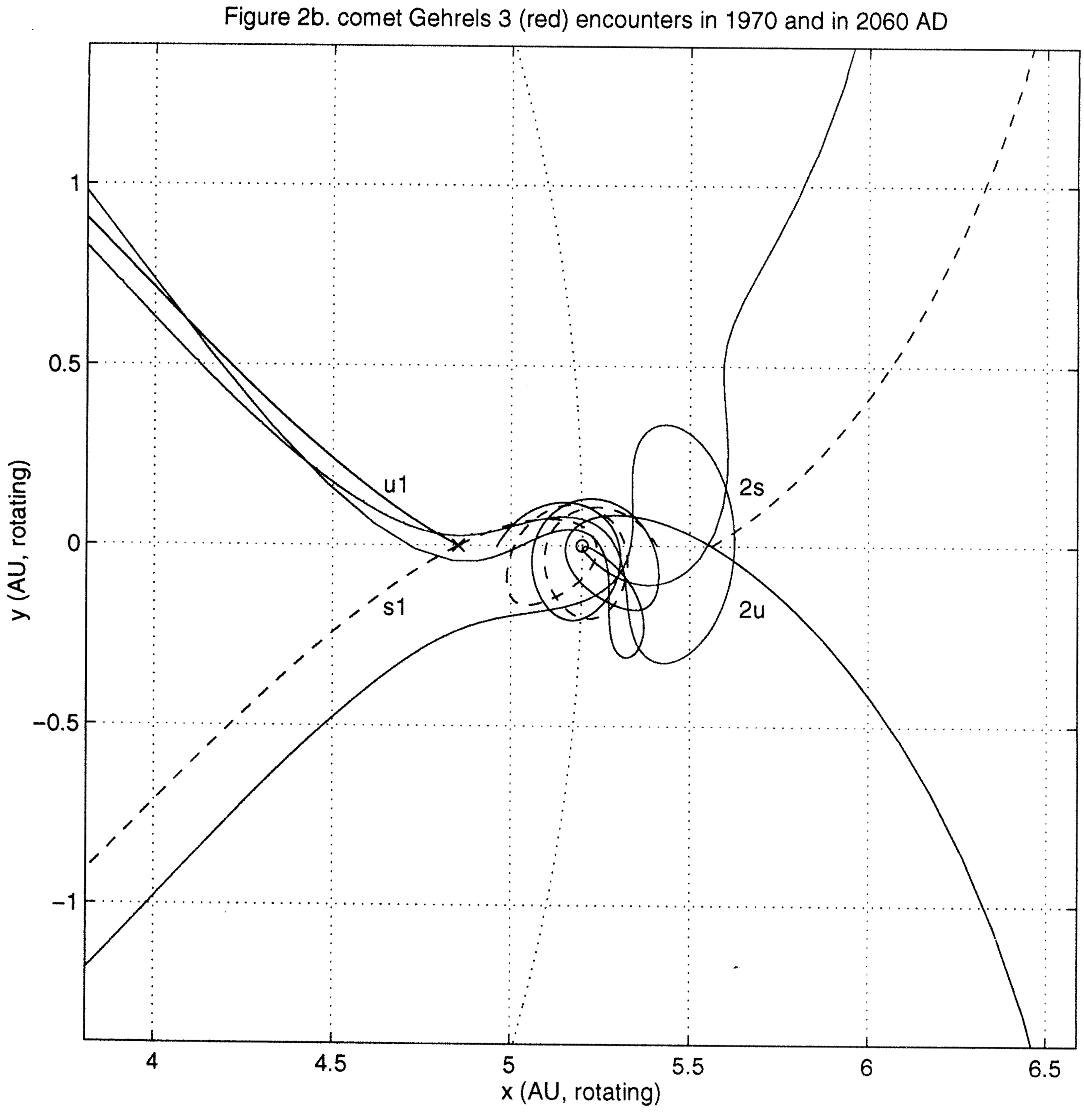


Figure 2b. comet Gehrels 3 (red) encounters in 1970 and in 2060 AD
y (AU, rotating)
x (AU, rotating)
1
0.5
0
-0.5
-1
4
4.5
5
5.5
6
6.5
u1
s1
2s
2u

Figure 3a. comet Oterma (red) from 1800–2000 AD with Jup us1 (green) and 2su (black)

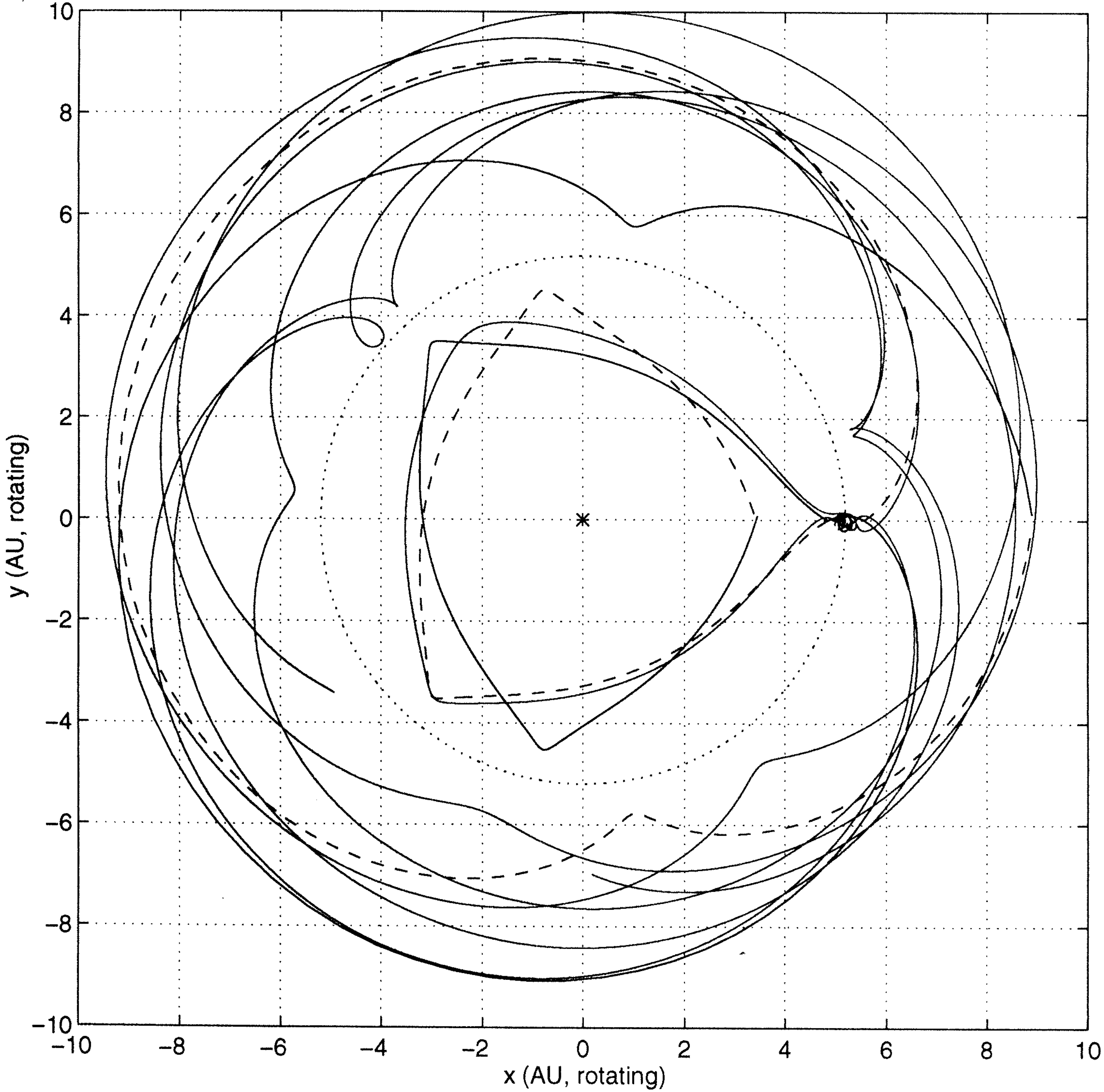
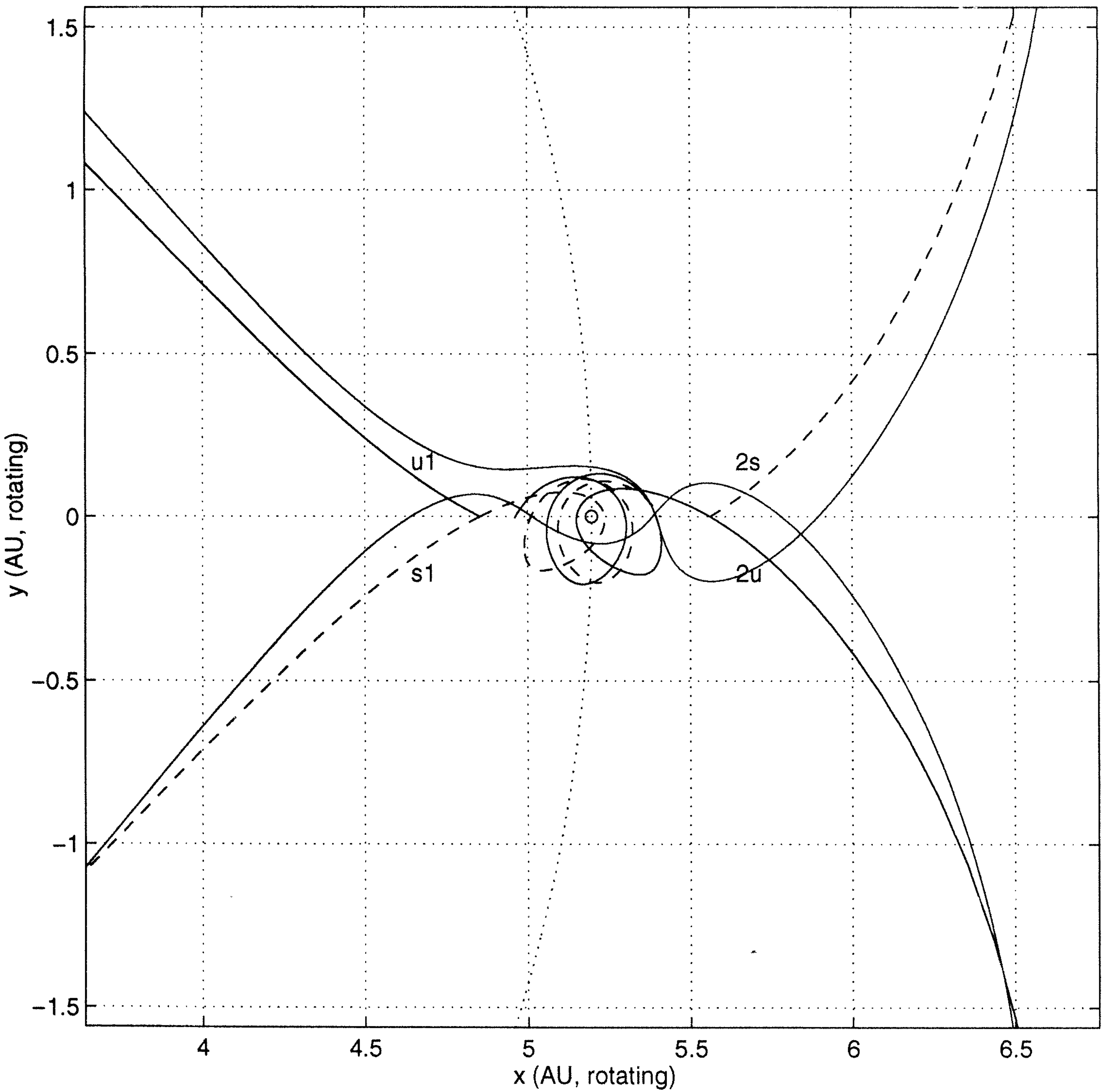
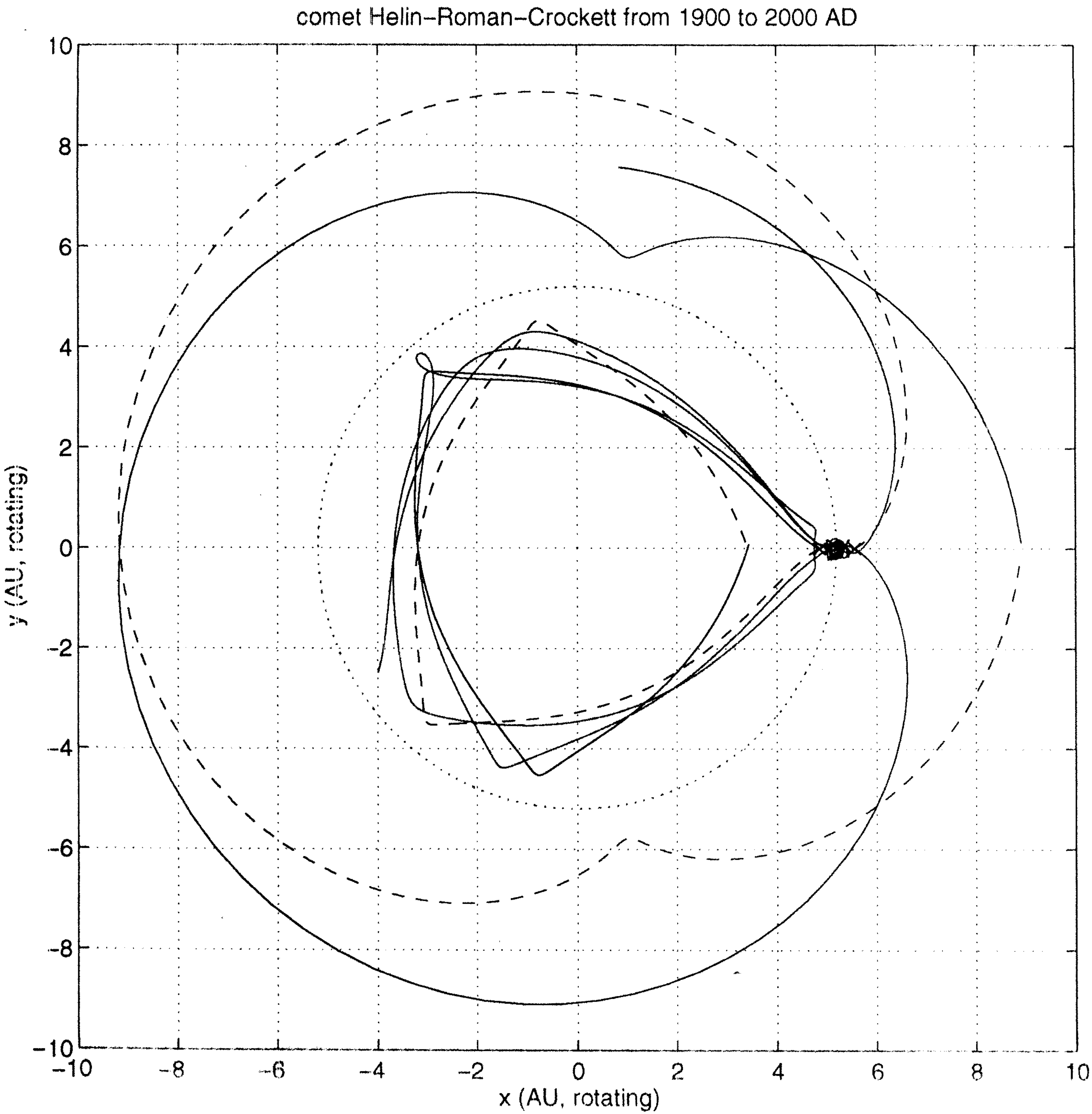


Figure 3b. comet Oterma (red) passing through capture region of Jupiter




comet Helin–Roman–Crockett from 1900 to 2000 AD
y (AU, rotating)
x (AU, rotating)
10
8
6
4
2
0
−2
−4
−6
−8
−10

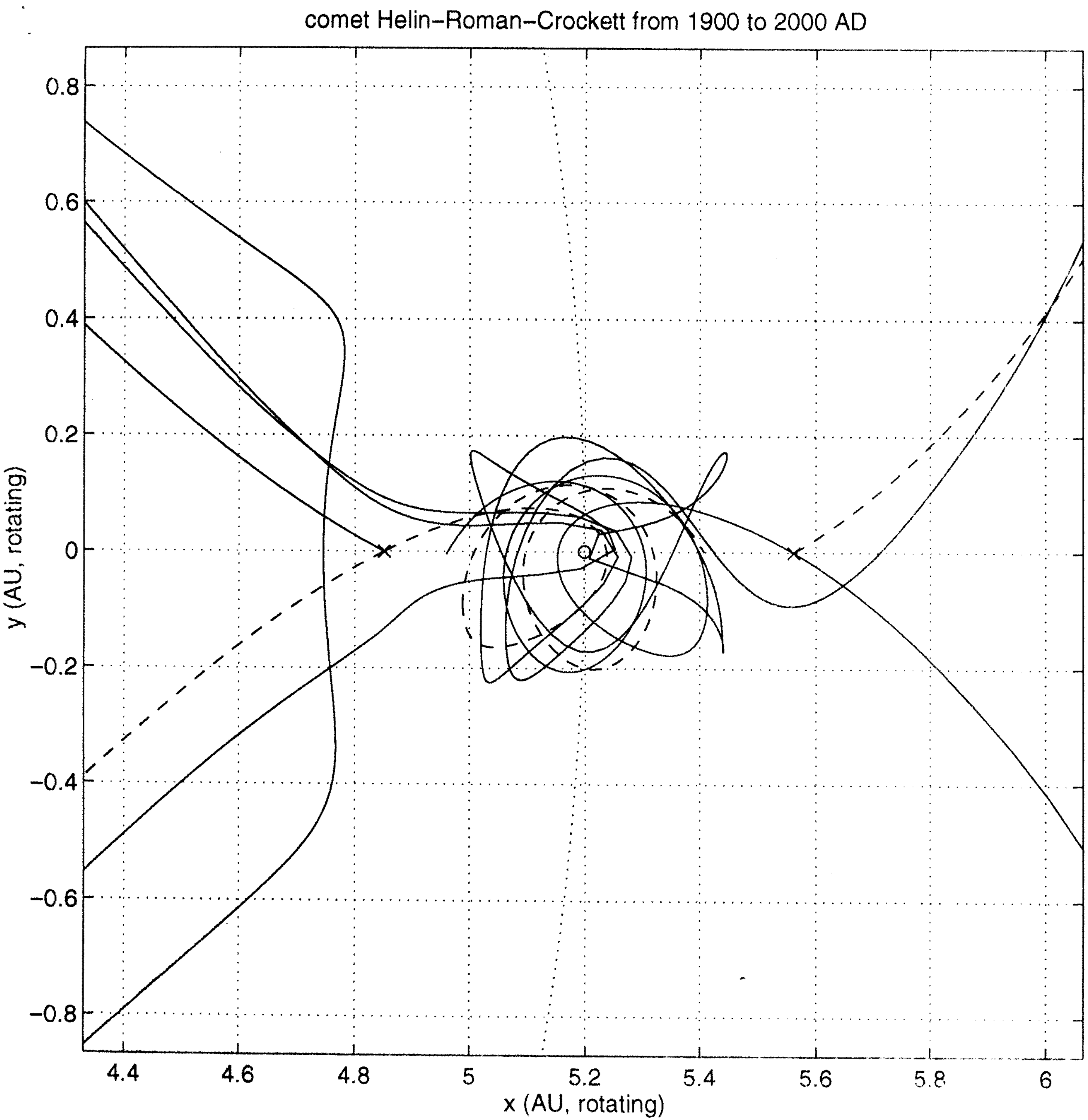

comet Helin–Roman–Crockett from 1900 to 2000 AD
y (AU, rotating)
x (AU, rotating)
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
4.4
4.6
4.8
5
5.2
5.4
5.6
5.8
6

Figure 5

Jupiter to Saturn transfer (min delV=~~986~~ 900 m/s at 8.36 AU)